

LIST OF ABSTRACTS

- INMACULADA BALDOMÁ BARRACA

Chaoticity of generic analytic convex billiards

This work deals with the abundance of analytic billiards having chaotic motions. The pioneering work by Zehnder's on planar twist maps in the 1970's was the first to provide a methodology for constructing analytic perturbations of maps in order to obtain transversality between the invariant manifolds of hyperbolic periodic orbits. In this work, we prove that the set of analytic billiards with negative curvature having a transversal homoclinic orbit to periodic orbits of any rational rotation number is generic in the usual analytic topology. In other words, we prove that, for analytic billiards, the coexistence of chaotic dynamics with periodic orbit of any period is prevalent. We use the Aubry-Mather theory to face with the transversality of periodic orbits away from the billiard's table boundary. This is a joint work with Anna Florio, Martin Leguil and Tere M-Seara.

- MISHA BIALY

Integrable billiards and marked length spectrum rigidity

It was proved long ago by Karl-Friedrich Siburg that Mather β -function distinguishes the circular billiards among all convex billiards. In my talk I shall show how to use Mather β -function to recover the elliptic billiards among all C^2 smooth centrally symmetric billiards.

- KOSTIANTYN DRACH

Spectral rigidity for expanding maps of the circle

For a smooth expanding map f of the circle, the (*unmarked*) *Lyapunov spectrum* of f at level $n \geq 1$ is defined as the set of logarithms of multipliers along periodic orbits of period n . The union of all these sets over all levels is analogous to the set of lengths of all closed geodesics on negatively curved surfaces—the classical length spectrum. In the talk, I will outline a Lyapunov spectral rigidity result for expanding circle maps. Namely, I will show that a smooth expanding circle map f of degree $d \geq 2$, under certain assumptions on the sparsity of its Lyapunov spectrum, cannot be perturbed with an arbitrarily small smooth perturbation (depending on f) so that the Lyapunov spectrum stays the same at all levels. The proof uses the Whitney extension theorem, a quantitative Li-type theorem, and a novel iterative scheme. This is joint work with

Vadim Kaloshin.

- CORENTIN FIEROBE

On the existence of periodic invariant curves for analytic families of twist-maps and billiards

Famous KAM results state the persistence of invariant curves for small perturbations of discrete integrable dynamical systems. These curves constitute a large set (i.e. of positive measure) of all curves in the initial system, and on each one of them the system is conjugated to a rotation of Diophantine rotation number. More recent results about so-called integrable billiards and Birkhoffs conjecture underline the importance of invariant curves on which the dynamic is conjugated to a rational rotation: it appears that such curves capture the peculiarity of a system and thus are important to study rigidity questions. In this talk, I will present a result obtained jointly with Alfonso Sorrentino on the persistence of such curves in analytic families of twist maps, and which can notably be applied to different billiard models. It extends a result by Arnaud, Massetti and Sorrentino.

- VASSILI GELFREICH

Billiards with slowly moving walls: ergodicity vs non-ergodicity

In this talk we discuss the role of ergodicity in the equilibration in slow-fast systems. We revise the equipartition law of the classical statistical mechanics. The derivation of the equipartition law relies on the ergodicity of the underlined dynamics. On the other hand the ergodic averaging theory predicts that in the presence of separated time scales the ergodicity of the fast sub-system impedes the equilibration of the whole system due to the presence of adiabatic invariants. We show that the violation of ergodicity in the fast dynamics effectively drives the whole system to the statistical equilibrium. We use models based on non-autonomous billiard to illustrate the ideas. This talk is based on joint works with V. Rom-Kedar, K. Shah and D. Turaev.

- COLIN GUILLARMOU

Marked length spectrum rigidity for surfaces with Anosov geodesic flows

We show that the marked length spectrum on surfaces with Anosov geodesic flows determine the metric up to isometry. Joint work with Lefeuvre and Paternain.

- EDMOND KOUDJINAN

On the Birkhoff conjecture for nearly centrally symmetric domains

In this talk, I will discuss a recent advance on Birkhoff conjecture, namely a proof that: an integrable, nearly centrally symmetric Birkhoff billiard table is necessarily an ellipse. This is done by combining recent breakthroughs by Bialy-Mironov (who prove the conjecture for centrally symmetric Birkhoff billiard tables) and by Kaloshin-Sorrentino (who prove the conjecture for Birkhoff billiard tables close to ellipses). In particular, we shall discuss the use of a nonstandard generating function discovered by Bialy-Mironov. Based on joint work with V. Kaloshin and Ke Zhang.

- ILLYA KOVAL

Billiard tables with analytic Birkhoff normal form are generically Gevrey divergent

The problem of the existence of an analytic normal form near an equilibrium point of an area-preserving map and analyticity of the associated coordinate change is a classical problem in dynamical systems going back to Poincaré and Siegel. One important class of examples of area preserving maps consists of the collision maps for planar billiards. Recently, Treschev discovered a formal bi-axially symmetric billiard with locally linearizable dynamics and conjectured its convergence. Since then, a Gevrey regularity for such a billiard was proven by Q. Wang and K. Zhang, but the original problem about analyticity still remains open. We extend the class of billiards by relaxing the symmetry condition and allowing conjugacies to non-linear analytic integrable normal forms. To keep the formal solution unique, odd table derivatives and the normal form are treated as parameters of the problem. We show that for the new problem, the series of the billiard table diverge for general parameters by proving the optimality of Gevrey bounds. The general parameter set is prevalent (in a certain sense has full measure) and it contains an open set. In order to prove that on an open set Taylor series of the table diverges we define a Taylor recurrence operator and prove that it has a cone property. All solutions in that cone are only Gevrey regular and not analytic.

- YUNZHE LI

Minimax geodesic and the spectral rigidity of Liouville metrics

We discuss some variational methods for constructing closed geodesics on the 2-torus, and apply these methods to study the spectral rigidity of Liouville metrics, i.e. metrics on 2-torus of the form $(f(x) + g(y))(dx^2 + dy^2)$. In contrast to the well-understood properties of minimal geodesics on 2-torus, we give examples to illustrate peculiar behaviours of non-minimal geodesics obtainable by these methods. This is part of a joint work with Joscha Henheik and Vadim Kaloshin.

- PAU MARTÍN

Parabolic saddles and Newhouse domains in celestial mechanics

McGehee introduced a compactification of the phase space of the restricted 3-body problem by gluing a manifold of periodic orbits “at infinity”. Although from the dynamical point of view these periodic orbits are parabolic (the linearization of the Poincaré map is the identity matrix), one of them, denoted here by O , possesses stable and unstable manifolds which, moreover, separate the regions of bounded and unbounded motion. This observation prompted the investigation of the homoclinic picture associated to O , starting with the work of Alekseev and Moser. We continue this research and extend, to this degenerate setting, some classical results in the theory of homoclinic bifurcations. More concretely, we prove that there exist Newhouse domains \mathcal{N} in parameter space (the ratio of masses of the bodies) and residual subsets $\mathcal{R} \subset \mathcal{N}$ for which the homoclinic class of O has maximal Hausdorff dimension and is accumulated by generic elliptic periodic orbits. One of the main consequences of our work is the fact that, for a (locally) topologically large set of parameters of the restricted 3-body problem the union of its elliptic islands forms an unbounded subset of the phase space and, moreover, the closure of the set of generic elliptic periodic orbits contains hyperbolic sets with Hausdorff dimension arbitrarily close to maximal. Other instances of the restricted n -body problem such as the Sitnikov problem and the case $n = 4$ are also considered.

- RAFAEL RAMÍREZ ROS

Chaotic properties of billiards in circular polygons

Circular polygons are closed C^1 strictly convex curves formed by finitely many circular arcs. There is a set in the phase space, corresponding to sliding trajectories close enough to the boundary of the domain, in which the return billiard dynamics is semiconjugate to a transitive subshift on infinitely many symbols with infinite topological entropy. There are billiard trajectories whose reflection angle after n impacts is $O(1/n)$ when $n \rightarrow +\infty$. The number of q -periodic billiard trajectories grows exponentially as $q \rightarrow \infty$. The length spectrum does not satisfy the classical Marvizi-Melrose asymptotic expansions. These results can be generalized to more general classical, dual and symplectic billiards. (This is a joint work with Andrew Clarke.)

- FRANK TRUJILLO

Inverse Problems in Analytic KAM Theory

According to classical KAM theory, a sufficiently small perturbation of a non-degenerate integrable Hamiltonian system admits a collection of invariant tori, whose restricted dynamics are conjugate to those of a rotation by a Diophantine vector. In this talk, we will discuss the following inverse problem: To what extent are the perturbed systems determined by their associated collections of invariant tori? We shall see that this collection completely characterizes the perturbed Hamiltonian, and show

some of the dynamical implications on systems sharing large collections of invariant tori.

- DANIEL TSODIKOVICH

Billiard tables with rotational symmetry

A simple geometric fact is that the only centrally symmetric, convex width, planar domain is a circle. This fact can be given an interpretation in terms of billiards, and then generalized: we show that for any k , any smooth domain with rotational symmetry of order k and an invariant curve of k -periodic orbits is a circle. Furthermore, we extend this result to other types of billiard systems: outer, symplectic, and Minkowski. This way we get rigidity results for all of those billiard systems. Joint work with Misha Bialy.

- DMITRY TURAEV

Adiabatic acceleration of a quantum-mechanical particle

We discuss how a perfectly adiabatic evolution of a quantum particle confined in a billiard domain with periodically and slowly oscillating boundaries can lead to an exponentially fast growth of particle energy.

- OTTO VAUGHN OSTERMAN

Length Spectrum Rigidity in Dispersing Billiard Systems

The problem of spectral rigidity in dispersing billiard systems is that of determining whether the set of perimeters of periodic orbits uniquely determines the billiard table up to isometry. We consider this problem for the class of dispersing billiard systems consisting of three scatterers in the plane satisfying the non-eclipse condition. My result is that the perimeters of a particular set of periodic orbits for two such systems are identical if and only if their collision maps are analytically conjugated to each other in some neighborhood of a particular homoclinic orbit.

- KE ZHANG

Determining the conjugacy class of symmetric Birkhoff billiards from length data

We show that the conjugacy class of a generic analytic Birkhoff billiard with a symmetry interchanging the two points on its diameter can be determined by the length data of a class of periodic orbits. Our results are inspired by the work in dispersing billiard by Balint, De Simoi, Kaloshin, and Leguil, and is a direct counter part to the recent result by O. V. Osterman for dispersing billiards. This is a joint work with V. Kaloshin and M. Leguil.